

Cyclic Universe with Quintom matter in Loop Quantum Cosmology

Hua-Hui Xiong^{1*}, Taotao Qiu^{1†}, Yi-Fu Cai^{1‡}, Xinmin Zhang^{1,2§}

¹ *Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918-4, Beijing 100049, P. R. China*

² *Theoretical Physics Center for Science Facilities (TPCSF), CAS, P. R. China*

Abstract

In this paper, we study the possibility of model building of cyclic universe with Quintom matter in the framework of Loop Quantum Cosmology. After a general demonstration, we provide two examples, one with double-fluid and another double-scalar field, to show how such a scenario is obtained. Analytical and numerical calculations are both presented in the paper.

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*xionghh@mail.ihep.ac.cn

†qiutt@mail.ihep.ac.cn

‡caiyf@mail.ihep.ac.cn

§xmzhang@mail.ihep.ac.cn

1 Introduction

Quintom model of dark energy [1] has a salient feature that its equation-of-state (EoS) crosses smoothly over the cosmological constant barrier $w = -1$. Phenomenologically, this kind of model is mildly favored by the current observational data fitting [2], but theoretically, the model building of Quintom dark energy is a challenge due to the No-Go theorem [3] (also see Refs. [1, 4, 5, 6, 7, 8]). This No-Go theorem forbids a traditional scalar field model with Lagrangian of the general form $\mathcal{L} = \mathcal{L}(\phi, \partial_\mu \phi \partial^\mu \phi)$ from having its EoS across the cosmological constant boundary. Consequently, in order to realize a viable Quintom model in the framework of Einstein's gravity theory, we need to add extra degrees of freedom to the conventional single field theory. The simplest Quintom model involves two scalars with one being Quintessence-like and another Phantom-like [1, 9]. This model has been studied in detail in [10, 11]. In recent years there have been a lot of theoretical studies of Quintom-like models. For example, motivated from string theory, the authors of Ref. [12] realized a Quintom scenario by considering the non-perturbative effects of a DBI action. Moreover, there are models which involve higher derivative terms for a single scalar field [13], models with vector field [14], making use of an extended theory of gravity [15], non-local string field theory [16], and others (see e.g. [17, 18, 19]). Due to this property, Quintom scenario has some interesting applications in cosmology. For example, a recent study has shown that a universe dominated by Quintom matter can provide a bouncing cosmology which allows us to avoid the problem of the initial singularity. Quintom Bounce, proposed in [20] with its perturbation developed in [21], takes place in the early time of the universe when the corresponding energy scale becomes very high. Therefore it may be accompanied by quantum effects of gravity.

One possible approach to investigating quantum gravity effects in the early universe is Loop Quantum Cosmology (LQC) [22] which is based on the theory of Loop Quantum Gravity (LQG) [23]. Beyond the usual knowledge of spacetime, LQC predicts that the underlying geometry is discrete and the big bang singularity is replaced by a big bounce when the energy density of the universe approaches the Planck scale [24, 25]. Therefore, the quantum geometry plays a significant role on determining the evolution of the universe in the early time. Interestingly, one notes that effects of loop quantum gravity also affect the evolution of Quintom universe in late time [26], since the energy density of phantom component usually increases during the expansion of the universe. In this paper we extend the idea of Quintom Bounce by considering a universe filled with Quintom matter in the frame of LQC. We interestingly find that the universe possesses an exactly cyclic evolution such that its scale factor undergoes contracting and expanding periodically.

In the context of LQC there exist two distinct quantum geometry modifications, namely the inverse volume modification [27, 28] and the quadratic density correction [25]. Based on the inverse volume modification authors of Ref. [29] obtained an cosmological solution with oscillating behavior for the closed universe (see Ref. [30] for cyclic universe in flat model). The inverse volume modification appears by choosing a large SU(2) representation for the holonomy in the matter part of Hamiltonian constraint. As pointed out in [28], it is natural to take the same spin representation for the gravitational sector as for matter sector in the whole Hamiltonian constraint. In this paper, we work with the fundamental spin representation both for the matter sector and gravity sector. So, the quantum geometry modification only comes from the quadratic density correction which arises from the “minimal area gap” in LQG. In this paper we consider quadratic density correction of LQC for the spatially flat universe.

This paper is organized as follows. In Sec. II, we study the possibility of building the models of the cyclic universe. Firstly, we will review briefly on LQC, then based on the modified Friedmann equation

from LQC we will present the general picture of the cyclic universe, followed by two examples. The last section is the discussion and conclusion.

2 Cyclic Universe with Quintom Matter

2.1 Loop quantum cosmology

LQG is a non-perturbative and background independent approach to quantizing classical gravity canonically. LQC inherits the framework developed from LQG, and so describes a quantized isotropic universe. In LQG, the phase space of the classical general relativity is described by $SU(2)$ connection A_a^i and densitized triads E_i^a . Considering a homogeneous and isotropic universe, such a symmetry of spacetime reduces the phase space of infinite degrees of freedom to be finite. Therefore, in LQC the classical phase space consists of the conjugate variables of the connection c and triad p , which satisfy Poisson bracket $\{c, p\} = \frac{1}{3}\gamma\kappa$, where $\kappa = 8\pi G$ (G is the gravitational constant) and γ is the Barbero-Immirzi parameter which is fixed to be $\gamma \approx 0.2375$ by the black hole thermodynamics. For the model of flat universe, the new variables obey the relations:

$$c = \gamma \dot{a}, \quad p = a^2, \quad (1)$$

where a is the scale factor of Friedmann-Robertson-Walker (FRW) universe. In terms of the connection and triad, the classical Hamiltonian constraint is given by [31]

$$\mathcal{H}_{cl} = -\frac{3}{\kappa\gamma^2}\sqrt{p}c^2 + \mathcal{H}_M. \quad (2)$$

As the same as in LQG there is no operator corresponding to connection c . For quantization the elementary variables are triad and holonomies of connection along an edge which is defined as $h_i(\mu) = \cos(\mu c/2) + 2\sin(\mu c/2)\tau_i$, where μ is the length of the i th edge with respect to the fiducial metric, and τ_i is related to Pauli matrices. The holonomies and the triads have well defined quantum operators such that for quantization the Hamiltonian constraint must be reformulated as the elementary variables, i.e., the holonomies and triad. In order to intimately mimic the quantization procedure of the full theory (LQG), the Hamiltonian constraint for quantization takes the full form as done in LQG [31], which can be reduced to the Eq. (2). The Hamiltonian constraint operator can be obtained by promoting the holonomies and triad to the corresponding operators. After quantization, the underlying geometry in LQC is also discrete as in the full theory, and the quantum difference equation governs the evolution of the universe. The quantum difference equation incorporating this discreteness can evolve through the “big bang point” without singularity.

Both in the improved framework of LQC and its original version, the semiclassical state is constructed, the results show that by evolving the semiclassical state backward in the expanding universe on the order of Planck scale the universe is bounced into a contracting branch [24, 25]. Furthermore, recently the bouncing behavior of LQC is confirmed in a solvable model [32]. It also indicates that the quantum feature of the universe can be well described by the effective theory which predicted the modified Friedmann equation. For the effective theory with the length scale larger than the Planck length the spacetime recovers the continuum, and the dynamical equation takes the usual differential form.

The effective Hamiltonian constraint is given by [33]

$$\mathcal{H}_{eff} = -\frac{3}{\kappa\gamma^2\bar{\mu}^2}\sqrt{p}\sin^2(\bar{\mu}c) + \mathcal{H}_M, \quad (3)$$

where $\bar{\mu}$ is the edge length of the square loop along which the holonomies are computed. The physical area of the square loop is fixed by the minimal area eigenvalue in LQG and is given by $\bar{\mu}^2 p = \alpha \ell_{\text{Pl}}^2$ (α is of order one and $\ell_{\text{Pl}} = \sqrt{G\hbar}$ with \hbar the reduced Planck constant). Here, \mathcal{H}_M is expressed by matter Hamiltonian which takes the same expression as its classical form. By the Hamiltonian constraint (3) one can get the Hamiltonian equation

$$\dot{p} = \{p, \mathcal{H}_{eff}\} = -\frac{\gamma\kappa}{3} \frac{\partial}{\partial c} \mathcal{H}_{eff} . \quad (4)$$

Squaring the above equation and making use of the weakly vanishing Hamiltonian constraint $\mathcal{H}_{eff} \approx 0$, the modified Friedmann equation can be obtained as

$$H^2 = \frac{\kappa}{3} \rho \left(1 - \frac{\rho}{\rho_c}\right) , \quad (5)$$

where $\rho_c = \frac{3}{\kappa\gamma^2\bar{\mu}^2 a^2} = \frac{3}{\kappa\gamma^2\alpha\ell_{\text{Pl}}^2}$, $\rho = \frac{\mathcal{H}_M}{a^3}$, and $H = \dot{a}/a$ is the Hubble parameter. As analyzed in [34], the modified Friedmann equation predicts a nonsingular bounce when the matter density approaches the critical value ρ_c .

In the context of the effective Hamiltonian, the matter Hamiltonian equations behave as its classical ones, so the energy conservation equation is still satisfied

$$\dot{\rho} + 3H(\rho + P) = 0 . \quad (6)$$

2.2 The general picture of cyclic universe

To start, we shall analyze in general how a cyclic universe works with Quintom matter in the framework of LQC. From Eq. (5), we can read that the Hubble parameter H happens to be zero when $\rho = \rho_c$. Furthermore, we can take the differentiations of both sides of Eq. (5) with respect to the cosmic time, and using Eq. (6), and then have

$$\dot{H} = -\frac{\kappa}{2} \left(1 - \frac{2\rho}{\rho_c}\right) (1+w)\rho . \quad (7)$$

From the expression above we can see that there is $\dot{H} = \frac{\kappa}{2}(1+w)\rho_c$ when $\rho = \rho_c$. So at this point, if $w < -1$, we have $\dot{H} < 0$ and thus a turnaround happens; on the other hand if $w > -1$, we have $\dot{H} > 0$ and correspondingly a bounce occurs.

The Quintom matter contains both quintessence- and phantom-like components, and for the former, the energy density grows in the contracting phase and decays in the expanding phase, while for the latter, the case is opposite ¶. So let's assume the universe is expanding at the beginning without losing generality. The quintessence-like component is decaying while phantom-like component is growing and gradually dominating the universe, making the total energy density growing and total EoS less than -1 . When the total energy density reaches the critical value ρ_c , the turnaround happens. The universe ceases expanding and turns to a contracting phase. In this phase, phantom-like component shows the decaying behavior and quintessence-like component becomes growing and finally dominating the universe which gives rise to the possibility that its energy density arrives at the critical point $\rho = \rho_c$ again. However, opposite to the case of expanding phase the EoS of the universe becomes larger than -1 this time, and hence a bounce takes place with the universe reentering the expanding phase. To conclude, a Quintom universe in LQC can evolve cyclically.

In the following section, we provide two examples of Quintom matter which can realize a cyclic universe in the framework of LQC.

¶The two components are related with a cosmic duality [11].

2.3 Example I: two-fluid Quintom matter in LQC

At first, we consider the case of Quintom that consists of two perfect fluid. For simplicity, we take both the two components to have constant EoS, with one being larger than -1 and the other being less than -1 . The total EoS can cross -1 occasionally, which is decided by the evolution of both components. So according to the energy conservation equation, the total energy density of this kind of Quintom should be $\rho = \rho_1 + \rho_2 = \rho_{1_0} a^{-3(1+w_1)} + \rho_{2_0} a^{-3(1+w_2)}$, where the subscript “1” and “2” denote quintessence-like and phantom-like component respectively and “0” stands for the initial value.

We begin our study in one of the expanding phase, just as where we are standing now. Since a is expanding, ρ_1 is decreasing and ρ_2 is increasing, and after a period of evolution the phantom-like component will dominate the universe. When the energy density reaches the critical value ρ_c , as discussed above, the Hubble parameter will decrease to zero, and then change from above zero to below zero, thus turnaround happens, and the universe turns into a contracting phase. In this phase, however, as the scale factor is contracting, ρ_1 will be increasing and ρ_2 will be decreasing, and from the same logic, the universe will be dominated by the quintessence-like component. When the total energy density reaches ρ_c again, Hubble parameter will vanish and a bounce occurs, which drives the universe into another expanding phase. Along this way, the expanding and contracting phase will take place alternately, giving the scenario of a cyclic universe.

In Fig. 1, we plot the solution of the numerical calculation. From the figure, we can see that this kind of matter has an oscillating EoS w around -1 , being a Quintom matter. Moreover, its energy density oscillates between a small positive value and ρ_c . Driven by it, the scale factor of the universe (as well as the Hubble parameter) oscillates gracefully, showing a cyclic universe scenario.

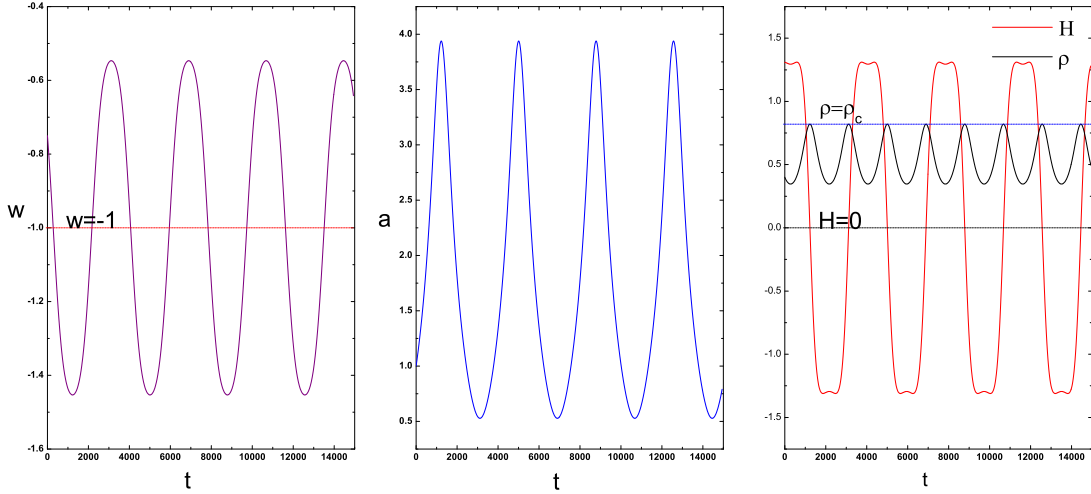


Figure 1: Plots of the evolution of the EoS parameter w , scale factor a , Hubble parameter H and energy density ρ in two-fluid Quintom model. In the numerical calculation we take $w_1 = -0.5$, $w_2 = -1.5$ and choose the initial values of parameters to be $\rho_{1_0} = 0.3$, $\rho_{2_0} = 0.1$, $a = 1.0$.

2.4 Example II: double-field Quintom matter in LQC

We now consider the Quintom model with two scalar fields. In spatially flat FRW cosmology, the line element is

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) . \quad (8)$$

We take the Lagrangian of the Quintom to be as:

$$\begin{aligned} \mathcal{L} &= \sqrt{-g} \left(\frac{1}{2} \dot{\phi}^2 - \frac{1}{2} \dot{\psi}^2 - V(\phi, \psi) \right) \\ &= \frac{1}{2} a^3 \dot{\phi}^2 - \frac{1}{2} a^3 \dot{\psi}^2 - a^3 V(\phi, \psi) , \end{aligned} \quad (9)$$

where we will choose the form of potential to be $V(\phi, \psi) = V_0[1 + \cos(\lambda\phi\psi)] + \frac{1}{2}m_1^2\phi^2 - \frac{1}{2}m_2^2\psi^2$. In [1] (see also [10]), it has already been proven that it is the simplest Quintom model. Here, for the cyclic universe the potential is taken to be interacting form. The conjugate momenta are given by

$$\begin{aligned} \Pi_\phi &= \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = a^3 \dot{\phi} , \\ \Pi_\psi &= \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = -a^3 \dot{\psi} . \end{aligned} \quad (10)$$

The whole Hamiltonian can be obtained as

$$\mathcal{H} = \frac{1}{2} a^{-3} \Pi_\phi^2 - \frac{1}{2} a^{-3} \Pi_\psi^2 + a^3 V(\phi, \psi) , \quad (11)$$

and the total energy density is

$$\rho = \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} \dot{\psi}^2 + V(\phi, \psi) . \quad (12)$$

According to the Hamiltonian Eq.(11) the equations of motion for the Quintom matter are

$$\ddot{\phi} + 3H\dot{\phi} = V_0\lambda\psi \sin(\lambda\phi\psi) - m_1^2\phi , \quad (13)$$

$$\ddot{\psi} + 3H\dot{\psi} = -V_0\lambda\phi \sin(\lambda\phi\psi) - m_2^2\psi . \quad (14)$$

In this model, there is an oscillating interaction term in the potential, which can cause the energy density oscillating to get a cyclic scenario. In our calculation, we found interestingly that by appropriately choosing the parameters in the potential, we can obtain an analytical solution. For example, if we choose $m_1 = m_2 = m$ and some appropriate initial conditions, we have

$$\phi = \phi_0 \sin(mt) , \quad \psi = \psi_0 \cos(mt) , \quad (15)$$

where ϕ_0 and ψ_0 are constants. Using these results, we can easily calculate the Hubble parameter H with the help of Eq.(5), which turns out to be:

$$H = H_0 \sin[\sin(2mt)] . \quad (16)$$

Though it is difficult to integrate H analytically to get scale factor $a(t)$, we can already read from the formula above that the Hubble parameter H is strictly periodical and can cross zero cyclicly, which means that the universe experiences expanding and contracting phase alternately. Fig.2 is our numerical results. Similar to the two-fluid case, we can see that in this case a cyclic universe is also obtained in the framework of LQC.

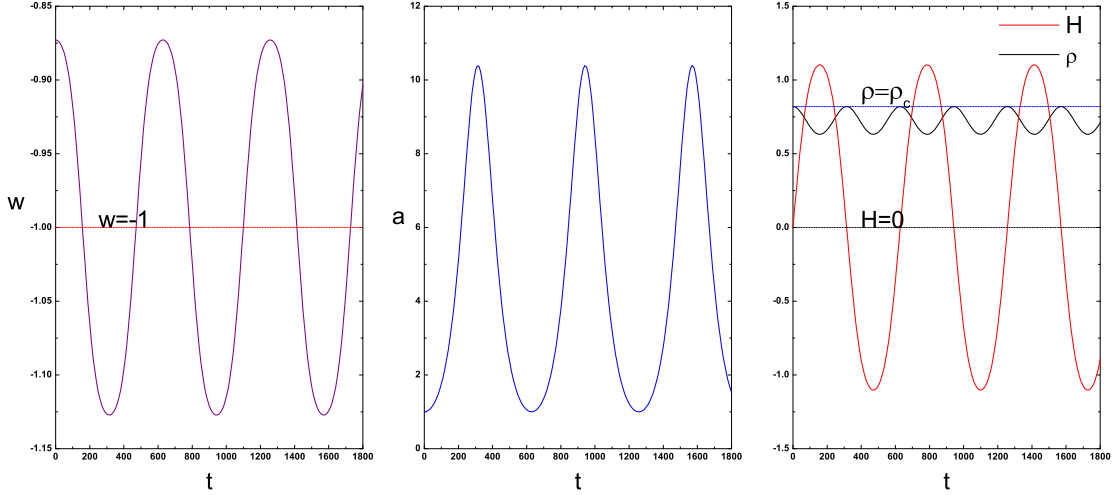


Figure 2: Plots of the evolution of the EoS parameter w , scale factor a , Hubble parameter H and energy density ρ for the Quintom model of two scalar fields. In the numerical calculation we take $m = 0.5$, $\lambda = 4.80$ and choose the initial values of parameters to be $\phi_0 = 0.42$, $\psi_0 = 0.42$, $V_0 = 0.41$.

3 Discussion and Conclusion

In this paper, we have studied an application of Quintom matter in the framework of LQC. Our results show that due to the specific property of Quintom and with the help of modified Friedmann equation, a scenario of cyclic universe can be realized naturally.

Before conclusion we should point out that:

1) in the scenario of cyclic universe with Quintom matter in LQC, there could be two possibilities causing the turnaround or the bounce, one being $\rho = \rho_c$ as studied in this paper, the other being $\rho = 0$ from Eq.(5). If the energy density reaches zero during the evolution, the turnaround or bounce can be realized as well which is pointed out in Ref. [20] (also see Ref. [44]). However, this possibility will not happen in the examples studied in this paper. For the double-fluid model, we can see from the formula that the energy density cannot approach zero; while for the double-field model, the “ghost” field ψ has a minus mass squared term, which prohibits the abnormal kinetic term to be so large to drive the total energy density to reach zero.

A cyclic universe is a non-standard scenario of cosmology which, by having the scale factor oscillating and the universe expanding and contracting alternately, is expected to solve the Big-Bang singularity and coincidence problem. In the literature there have been many discussions on such a topic and a number of models have been proposed, among which there are cyclic models in braneworld scenario [35, 36, 37], cyclic cosmologies with spinor matter [38], closed oscillating universe [29, 39], cyclic braneworld in LQC [30, 34], and see Refs. [26, 40] for recent developments, and so on. The difference of our model from these models is that our model is in the framework of 4-dimensional spatially flat FRW universe with LQC.

2) our model provides a qualitative picture of cyclic evolution of the universe. In this paper we have

not included the radiation and matter, however, the general behavior of our solution will not be changed since the epochs for radiation and matter only last for a short time in one period during the whole evolution. There are some open issues in the scenario of cyclic universe, such as the entropy problem [41, 42], blackhole problem [43, 44, 45], the problem of causality [41] and so on, which is still in discussion in the literature.

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